

# The NLO DGLAP extraction of $\alpha_s$ and higher twist terms from CCFR $xF_3$ and $F_2$ structure functions: results and scale dependence

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We perform a detailed NLO analysis of the combined CCFR  $xF_3$  and  $F_2$  structure functions data and extract the value of  $\alpha_s$ , parameters of distributions and higher-twist (HT) terms using a direct solution of the DGLAP equation. The value of  $\alpha_s(M_Z) = 0.1222 \pm 0.0048(\text{exp}) \pm 0.0026(\text{theor})$  is obtained. Our result has a larger central value and errors than the original one of the CCFR collaboration due to model independent parametrization of the HT contributions. The dependence of HT contributions on the QCD renormalization scale is studied.

In the recent years interest to the problem of the extraction of the high-twist-terms from the analysis of different deep-inelastic scattering (DIS) data was renewed, mainly due to the possibility to model these terms in different processes using the infrared-renormalon (IRR) technique (see e.g. Refs. [1]-[5] and, especially, Ref. [6] for the review).

On the other hand, the experimentalists improve their data precision and achieve, sometimes, a percent level of accuracy. For example, very precise data on  $xF_3$  and  $F_2$  from the  $\nu N$  DIS experiment, performed at Tevatron by the CCFR collaboration, recently appeared [7,8]. The CCFR data on  $xF_3$  were analyzed in Ref. [9] in the next-to-leading-order (NLO), and with an approximate next-to-next-to-leading order (NNLO) corrections. For the latter the NNLO QCD corrections to the coefficient function [10] were taken into account. The NNLO corrections to the anomalous dimensions of a limited set of even non-singlet moments [11] were also taken into account. The NNLO corrections to the anomalous dimensions of odd moments, which are not still explicitly calculated, were obtained using smooth interpolation procedure proposed in Ref. [12] and improved in Ref. [9]. The aim of Ref. [9] was to attempt the first NNLO determination of  $\alpha_s(M_Z)$  from DIS and to extract the HT terms from the data on  $xF_3$  within the framework of the IRR-model [3]. Alongside, the model-independent extraction of the HT terms was made, similarly to the analysis of the combined SLAC-BCDMS data [13], which was performed in the NLO approximation. Theoretical uncertainties of the analysis of Ref. [9] were further estimated in Refs. [14] in the N<sup>3</sup>LO approximation using the method of

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\*Supported in part by the Russian Foundation of Basic Research, Grant N 99-01-00091

Padé approximants. It has been found in Refs. [9,14] that the inclusion of the NNLO corrections leads to the decrease of the HT terms, so that at the NNLO its  $x$ -shape variation is closer to zero.

In this work we completed the fits of Ref.[15], performed the NLO analysis of the CCFR data on the structure functions  $F_2$  and  $F_3$  with the help of a QCD DGLAP evolution code, developed in Ref. [16]. It should be stressed that the code [16] was tested using the procedure proposed in Ref. [17] and demonstrated the accuracy at the level of  $O(0.1\%)$  in the kinematic region covered by the analyzed data. Our fits were made in the NLO approximation within the modified-minimal-subtraction ( $\overline{MS}$ ) factorization and renormalization schemes. The effective number of flavours  $n_f$  was chosen to be  $n_f = 4$  for  $Q^2$  less than the definite scale  $M_5^2$  and increased to  $n_f = 5$  at larger values of  $Q^2$  keeping the continuity of  $\alpha_s$  [19]. The value of the effective matching scale  $M_5$  was varied from  $M_5 = m_b$  to  $M_5 = 6.5m_b$ . The latter choice was advocated in Ref. [20] on the basis of the DIS sum rules consideration. The dependence of the fit results on the choice of the matching point gives one of the sources of theoretical uncertainties inherent to our analysis.

The leading twist terms  $xF_3^{LT}(x, Q)$  and  $F_2^{LT}(x, Q)$  were obtained by direct integration of the DGLAP equations for non-singlet, pure-singlet, and gluon distributions that were subsequently convoluted with the coefficient functions. In order to provide the straightforward way for comparison of our results with Ref. [16], the initial reference scale for pQCD evolution  $Q_0^2 = 9 \text{ GeV}^2$  was taken. The boundary conditions at this reference scale were chosen in the form analogous to the ones, used in Refs. [8,9]:

$$xq^{NS}(x, Q_0) = \eta_{NS}x^{b_{NS}}(1-x)^{c_{NS}}(1+\gamma x)\frac{3}{A_{NS}}$$

for non-singlet distribution,

$$xq^{PS}(x, Q_0) = \eta_Sx^{b_S}(1-x)^{c_S}/A_S$$

for pure-singlet distribution, and

$$xG(x, Q_0) = \eta_Gx^{b_G}(1-x)^{c_G}/A_G$$

for gluon distribution, where  $A_{NS}$ ,  $A_S$ , and  $A_G$  were defined from the partons' number/momentum conservation and other parameters were fitted.

The expression for the  $xF_3$  and  $F_2$  that includes the HT contribution looks as follows:

$$xF_3^{HT}(x, Q) = xF_3^{LT,TMC}(x, Q) + \frac{H_3(x)}{Q^2}, \quad F_2^{HT}(x, Q) = F_2^{LT,TMC}(x, Q) + \frac{H_2(x)}{Q^2},$$

where  $F_{2,3}^{LT,TMC}(x, Q)$  are  $F_{2,3}^{LT}(x, Q)$  with the target mass correction applied. We used the model independent HT-expression, i.e.  $H_{2,3}(x)$  were parametrized at  $x = 0., 0.2, 0.4, 0.6, 0.8$  with linear interpolation between these points.

The account of the point-to-point correlations of the data due to systematic errors can be crucial for the estimation of total experimental errors of the parameters (see, in particular, Ref. [18], where the value  $\alpha_s(M_Z) = 0.1180 \pm 0.0017$  (stat+syst) was obtained as a result of the combined fit to the SLAC-BCDMS data with HT included). The systematic errors were taken into account analogously to the works of Refs.[16,18]. The

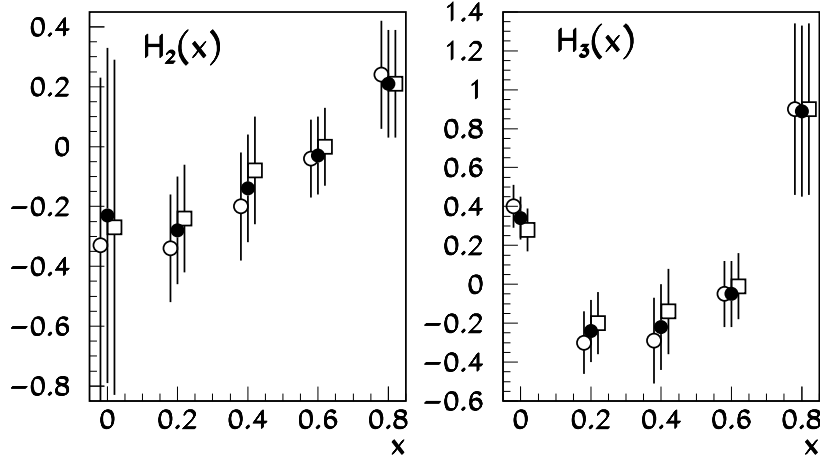


Figure 1. The high-twist contribution to the structure functions  $F_2$   $F_3$ . Full circles correspond to the fit with renormalization scale parameter  $k_R = 1$ , empty circles – to the fit with  $k_R = 1/4$ , squares – to the fit with  $k_R = 4$ .

total number of the independent systematic errors sources for the analyzed data is 18 and all of them were convoluted into a general correlation matrix, which was used for the construction of the minimized  $\chi^2$ . In addition to the point-to-point correlation of the data due to systematic errors, the statistical correlations between  $F_2$  and  $xF_3$  were also taken into account. The account of systematic errors leads to a significant increase of the experimental uncertainties of the HT parameters and the shift of their central values. However, even in this case, there is a definite agreement with the results on HT-behaviour of Ref. [9], obtained in NLO. Moreover, these results do not contradict the IRR-model prediction of Ref. [3], since fitting IRR model parameter  $A'_2$  to the data, we obtained  $A'_2 = -0.10 \pm 0.09 \text{ GeV}^2$ , compatible with  $A'_2 \approx -0.20 \text{ GeV}^2$  and  $A'_2 \approx -0.1 \text{ GeV}^2$ , given in Ref. [3] and Ref.[4].

Performing the trial fits we got convinced that the introduction of the factor  $(1 + \gamma x)$  into the reference expressions for the the gluon and singlet distributions does not improve the quality of the fit and does not change the value of  $\alpha_s$ . Also, we fixed parameters  $\gamma_{NS}$ ,  $b_S$  and  $b_G$  at zero because this increase the value of  $\chi^2$  by few units only while  $\chi^2/\text{NDP}$  remained less than unity.

The results of the fit on  $H_2(x)$  and  $H_3(x)$  parameters are given in Fig. 1. One can notice that, comparing with the fit to  $xF_3$  data only from Ref. [9], the HT parameters errors are decreasing. Within the errors, the parameters that describe the boundary distributions are compatible with ones of Ref. [8]. The  $H_3(x)$  coefficients are in agreement with the NLO results of Ref. [9] and the behaviour of  $H_2(x)$  qualitatively reproduce the HT contribution to  $F_2$  that was obtained from the combined fits to the SLAC-BCDMS data on  $F_2$  [13,18].

When the matching scale  $M_5$  was changed from  $m_b$  to  $6.5m_b$ , the value of  $\alpha_s(M_Z)$  shifted down by 0.0052 and, hence, the theoretical error in  $\alpha_s(M_Z)$  due to uncertainty of b-quark threshold can be estimated as 0.0026. This uncertainty is in agreement with the

results of the fits to the CCFR data obtained within the so-called spline  $\overline{MS}$  prescription [21] with the help of the Jacobi polynomial method [22]. One more source of the theoretical uncertainty due to the truncation of higher QCD orders was evaluated following the way, which was proposed in Ref. [13]. In accordance with their procedure, one can introduce renormalization scale  $k_R$  into QCD evolution equations in the way, given below for non-singlet evolution:

$$\frac{dxq^{NS}}{d\ln Q} = \frac{\alpha_s(k_R Q)}{\pi} \int_x^1 dz \left\{ P_{qq}^{NS,(0)}(z) + \right. \\ \left. + \frac{\alpha_s(k_R Q)}{2\pi} \left[ P_{qq}^{NS,(1)}(z) + \beta_0 P_{qq}^{NS,(0)}(z) \ln(k_R) \right] \right\} \frac{x}{z} q^{NS}(x/z, Q),$$

where  $P^{NS,(0)}$  and  $P^{NS,(1)}$  denote the LO and the NLO parts of the non-singlet splitting function. The dependence of the results on  $k_R$  would signal an incomplete account of the perturbation series. The shift of  $\alpha_s(M_Z)$  resulting from the variation of  $k_R$  from 1/4 to 4 turned out to be only 0.0007. At the same time one can observe a simultaneous variation of  $H_{2,3}(x)$  (see Fig. 1). This effect can denote, that, in fact, the fitted values of  $H_{2,3}(x)$  incorporate the higher order QCD contributions (NNLO and beyond, c.f. Ref. [9,14]). It is interesting, that the same effect was also observed in the analysis of charged leptons data [23], i.e. it cannot be attributed to a specific feature of the CCFR data. The interplay between these contributions and genuine power corrections does not allow for their unambiguous separation. The value of  $\alpha_s$  is strongly correlated with the fitted high twist, that leads to increase of the  $\alpha_s$  error and, consequently, in our fit with simultaneous determination of  $\alpha_s$  and high twist contributions the theoretical error in  $\alpha_s$  due to truncation of higher QCD orders is merged into the total experimental errors. Having taken  $Q_0^2 = 20 \text{ GeV}^2$  as an initial scale, we checked that our NLO results were quite stable to the variation of the initial scale. However, we do not know what will happen at the NNLO, where the stability to  $Q_0^2$  was observed only starting from  $Q_0^2 \approx 20 \text{ GeV}^2$  [14].

The final value of  $\alpha_s$  in NLO with the account of theoretical uncertainties is given as  $\alpha_s(M_Z) = 0.1222 \pm 0.0048$  (stat + syst)  $\pm 0.0026$  (theor.) It differs a bit from the NLO value  $\alpha_s(M_Z) = 0.119 \pm 0.002$  (stat+syst)  $\pm 0.004$  (theory) obtained in the CCFR analysis [7]. The increase of the experimental error is due to that CCFR group used model-dependent form of the HT contributions, while we considered them as the additional free parameters and extracted them from the fit.

It should be stressed, that the scale-dependence uncertainty of the NLO results drastically minimized to the value of 0.0007 after taking into account HT corrections. The decrease of this uncertainty was also recently found in the analysis without HT terms after taking into account 3-loop splitting function [24]. Thus we think that more careful NNLO DGLAP analysis of the DIS data with HT effects included is now on the agenda.

**Acknowledgements** One of us (ALK) would like to thank the Organizers of Nucleon99 Workshop for hospitality in Frascati and for the financial support.

## REFERENCES

1. M. Beneke and V.M. Braun, *Phys. Lett.* **B348** (1995) 513.
2. Yu. L.Dokshitzer, G. Marchesini and B.R. Webber, *Nucl. Phys.* **B469** (1996) 93.
3. M. Dasgupta and B.R. Webber, *Phys. Lett.* **B382** (1996) 273.
4. M. Maue, E. Stein, A. Schäfer and L. Mankiewicz, *Phys. Lett.* **B401** (1997) 100.
5. R. Akhoury and V.I. Zakharov, *Proc. of QCD-96 Int. Workshop, Montpellier*, ed. S. Narison, *Nucl. Phys. Proc. Suppl.* **54A** (1997) 217; (hep-ph/9610492)
6. M. Beneke, preprint CERN-TH-98-233 (hep-ph/9807443).
7. CCFR collaboration, W.G. Seligman et al., *Phys. Rev. Lett.* **79** (1997) 1213.
8. W.G. Seligman, Thesis, Report No. Nevis-292, 1997.
9. A.L. Kataev, A.V. Kotikov, G. Parente and A.V. Sidorov, *Phys. Lett.* **B388** (1996) 179; *Phys. Lett.* **417B** (1998) 374.
10. E.B. Zijlstra and W.L. van Neerven, *Nucl. Phys.* **B383** (1992) 235.
11. S.A. Larin, T. van Ritbergen and J.A.M. Vermaseren, *Nucl. Phys.* **B427** (1994) 41; S.A. Larin, P. Nogueira, T. van Ritbergen and J.A.M. Vermaseren, *Nucl. Phys.* **B492** (1997) 338.
12. G. Parente, A.V. Kotikov and V.G. Krivokhizhin, *Phys. Lett.* **B333** (1994) 190.
13. M. Virchaux and A. Milsztain, *Phys. Lett.* **B274**(1992) 221.
14. A.L. Kataev, G. Parente and A.V. Sidorov, Preprint IC/99/51 (hep-ph/9905310).
15. S.I. Alekhin and A.L. Kataev, *Phys. Lett.* **B452** (1999) 402.
16. S.I. Alekhin, IHEP 96-79, hep-ph/9611213; *Eur. Phys. J.* in print.
17. J. Blümlein et al., Report No. DESY 96-199, hep-ph/9609400; *Proc. "Hamburg 1995/1996, Future physics at HERA"*, 1996, p. 23.
18. S.I. Alekhin, *Phys. Rev.* **D59** (1999) 114016.
19. W. Bernreuther and W. Wetzel, *Nucl. Phys.* **B197** (1982) 228; *ibid.* **B513** (1998) 758 (Err.); S.A. Larin, T. van Ritbergen and J.A.M. Vermaseren, *Nucl. Phys.* **B438** (1995) 278; for the definite cross-checks see M. Spira, *Fortsch. Phys.* **46** (1998) 203; K.G. Chetyrkin, B.A. Kniehl and M. Steinhauser, *Phys. Rev. Lett.* **79** (1997) 2184.
20. J. Blümlein and W.L. van Neerven, *Phys. Lett.* **B450** (1999) 417.
21. D.V. Shirkov, A.V. Sidorov and S.V. Mikhailov, hep-ph/9707514
22. G. Parisi and N. Surlas, *Nucl. Phys.* **B151** (1979) 421; J. Chýla and J. Ramez, *Z. Phys.* **C31** (1986) 151; V.G. Krivokhizhin et al., *Z. Phys.* **C36** (1987) 51; *ibid.* **C48** (1990) 347.
23. S.I. Alekhin, hep-ph/9907350, to appear in the Proceedings of XXXIV Moriond conference "QCD and high energy hadronic interactions", March 20-27, 1999, Les Arcs.
24. W.L. van Neerven and A. Vogt, preprint INHLO-PUB-14/99 (hep-ph/9907472)